SIMULATION OF DYNAMIC PROCESSES IN NONLINEAR DISCRETE SYSTEMS WITH DELAY ON THE BASIS OF GRAPHS

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Abstract

In connection with the problems of development and implementation of perfect automatic and automated control systems for complex technological processes based on modern information technology tools, the development of research methods for multidimensional nonlinear discrete control systems is of particular relevance. The variety of these systems is due to the types of modulation, non-linearities, modes of operation of impulse elements, characteristics of continuous parts. The article deals with the problems of modeling and research of one-dimensional and multidimensional pulse-frequency systems with delay based on the use of dynamic graphs. One of the key points in solving the problem is an approach based on the maximum consideration of the physical features of these systems. Given the structural complexity of systems, it is necessary first of all to establish the possibility of decomposing the system into simpler subsystems. Then the study of the system can be reduced to the study of individual subsystems and the nature of their interaction, that is, to the determination of the properties of the system as a whole by the properties of its elements. Systems with frequency-pulse modulation are essentially non-linear systems. The topological method based on dynamic graph models allows solving the fundamental difficulties of studying nonlinear discrete systems with delay, allows clearly and strictly formalizing their description, and makes it possible to obtain effective algorithms for analyzing one-dimensional and multidimensional systems of this class. The article presents an example of using the obtained algorithm.

Keywords: *nonlinear discrete system, frequency pulse modulator, frequency-pulse system with delay, dynamic processes, graph model*

Introduction. Currently, an exceptionally digital technique is used to manage automatic systems of various nature and various purposes. Measurements and control are carried out in discrete times, therefore discrete mathematical models of dynamic systems are relevant. Initially, systems with pulse-frequency modulation were widely used in radio engineering and telemetry [1]. At present, pulse-frequency systems are widely used in information and measurement technology [2,3,4], in automatic control systems [5,6,7], in modulating installations [8] and in the study of nervous activity [9,10]. Pulse-frequency systems belong to the class of essentially nonlinear systems, since there is a nonlinear element in their structure. To date, several methods of analyzing pulse-frequency systems have been developed. The first one is based on precise calculation methods and includes questions of obtaining and analyzing difference equations describing the dynamics of these systems. The second direction is based on approximate methods related to the linearization of a nonlinear element. Frequency methods based on the direct method of A. M. Lyapunov and the frequency access of V. M. Popov have found wide use in the study of the dynamics of pulse-frequency systems. A detailed review of the work and analysis of methods for studying pulse-frequency systems is given in [11- 15].

Materials and methods. Fig. 1 shows a non-linear discrete system with a constant control delay τ. Nonlinearity is created by a pulse-frequency modulator (PFM); the object is represented by a dynamic link of the l-th order (Figure1).

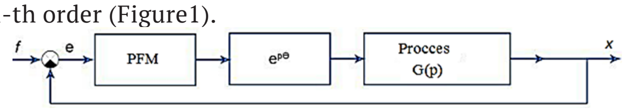

Figure 1. Nonlinear discrete system with constant control delay

The modulator generates a clock sequence of pulses according to the modulation characteristic.

$$T_j = t_{j+1} - t_j = \phi[e(t_j)], \qquad (1)$$

where is the error signal in the j-th interrupt period, t_j - is the moment of appearance of the j-th pulse at the output of the pulse-frequency modulator, T_j -is the variable interrupt period of the pulse-frequency modulator.

The output signal of the pulse-frequency modulator changes according to the formula:

$$e_j^*(t) = \begin{cases} signe(t_j), & t_j < t \le t_j + \tau_u \\ 0, & t_j + \tau_u < t \le t_{j+1} \end{cases}, \qquad (2)$$

On figure 2 proposes an equivalent circuit diagram of the system. The zero-order extrapolator in the frequency-pulse system keeps the signal $e^*(t_j)$ for a time equal to the duration of the frequency-modulated pulses τ_u.

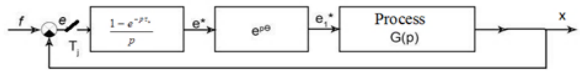

Figure 2. Equivalent circuit of a pulse-frequency system with a delay of the control signal

At the output of the frequency-pulse modulator, a sequence of pulses of the same width, but with a variable frequency, is formed [16–17]. The delay link delays this sequence by time θ (Figure 3).

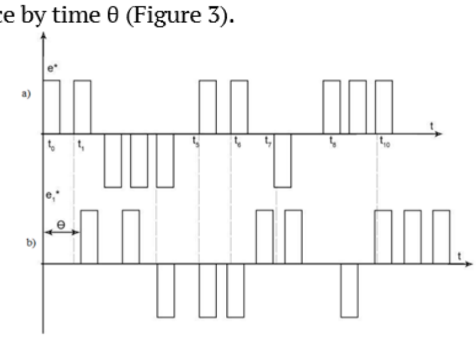

Figure 3. Pulse sequence at the output of the pulse - frequency modulator (a); pulse sequence at the output of the delay link (b).

Delayed control signals do not affect the linear part of the system under the condition - t ⩽ θ. For t ⩽ θ, the structural states of the system replace each other from cycle to cycle. In each of these states, the system is linear and open-loop. From the output of the pulse-frequency modulator, the control signals arrive at the linear part of the system, provided that t > θ.

At t> θ, there is a non-programmed change in the structure of the system. The structural states of the system change in an arbitrary sequence (Figure 4). But in each of them, the system is also a linear open system [18-19]. Let us proceed to the construction of macromodels of the dynamics of the functioning of a pulse-frequency system with a constant delay.

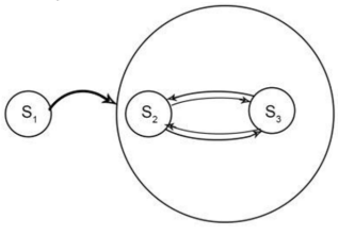

Figure 4. Changing the structure of a nonlinear discrete system with delay

Suppose that in the time interval [0, θ] the number of closures of the pulse-frequency modulator is equal to (k+1).

The sequence of pulses issued by the modulator at moments t0, t1,t2,..., tk is not skipped by the delay link.

Under the condition t > θ, some moment of closing the impulse element tq can be in different intervals. It depends on the value of the delay θ and the value of the variable interruption period of the pulse-frequency modulator (Figure 5).

a) within the range of the j-th pulse of the delayed pulse sequence, i.e.

$$t_j + \theta < t_q \le t_j + \theta + \tau_u$$

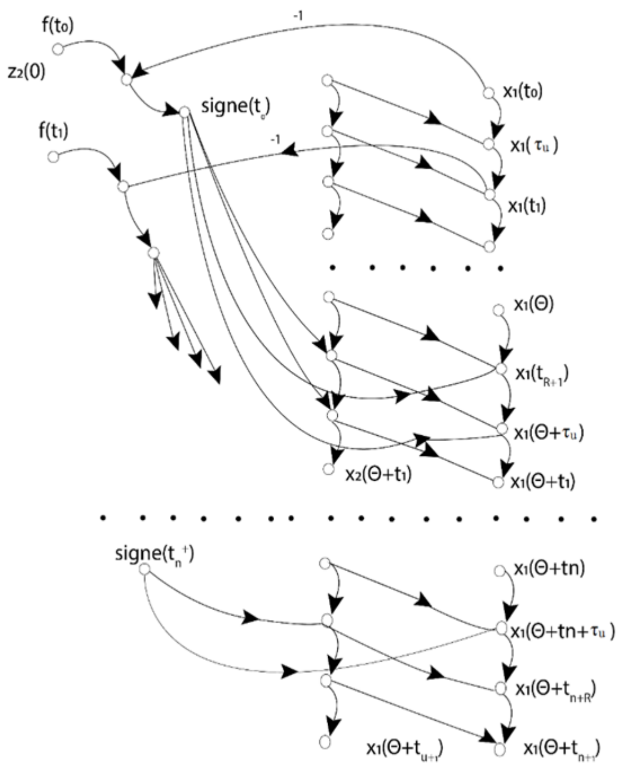

Figure 5. Dynamic graph model of the pulse - frequency system.

As can be seen from this condition, the state variables of the system are determined first for the moment of time tj+ θ, then for the moment of time tj+θ+τu ;

b) outside the zone of action of the j-th pulse of the delayed sequence, i.e. within the interval covering the pause between the j-th and (j + 1)-th pulses.

When this condition is met, the state variables of the system are determined first for the moment tj+θ, then for the moment tq, and so on.

$$t_j + \tau_u + \theta < t_q \le t_j + \theta.$$

In this case, the processes are defined first for the moment tj+θ+τu , then for the moment tq, and so on.

The dynamic graph model of a system with a pulse-frequency modulator is shown in Fig. 5. We write down the recurrent relations obtained from the analysis of graph models.

t ⩽ θ

j = 0, 1, ... , k-1,

$$e(t_j^+) = f(t_j) - x_1(t_j), \quad (1)$$

$$e^*(t_j^+) = signe(t_j^+), \quad (2)$$

Tj = tj+1 – tj = ψ [e(tj)], (3)

X(tj+τu) = A (τu) X(tj) (4)

X(tj+1) = A (Tj) X(tj) (5)

X(θ) = A (θ - tk) X(tk) (6)

t > θ

j = k, k+1, ..., n.,

$$e(t_j^+) = f(t_j) - x_1(t_j),$$

$$e^*(t_j^+) = signe(t_j^+),$$

Tj = ψ [e(tj)],

If tj-k + θ < tj+1 ≤ tj-k + θ + τu, then:

X(tj+1)=A(tj+1- α)X(α)+B(tj+1- α)sign e(t+j -k), (7)

X(α +τu)=A(τu) X(α)+B(τu) sign e(t+j- k), (8)

X(tj-k+1+θ) =A(Tj-k)X(α)+B(Tj-k)signe(t+j-k+1), (9)

where α = θ + tj-k

If tj -k + θ + τu < tj+1 < tj-k+1 + θ, then

X(α + τu) = A(τu) X(α) + B(τu) sign e(t+j -k), (10)

X(tj+1) = A(tj+1 - α -τu) X(α + τu), (11)

X(tj-k+1+θ) = A(Tj -k) X(α) + B(Tj -k) sign e(t+j -k+1). (12)

Definition of system state variables with pulse-frequency modulator and control signal delay

1. A dynamic graph model of the system is built - the subgraph of the delayed signal and the subgraph of the continuous part of the system are combined.

2. For all steps starting from the first one (i = 0, 1,..., k), we define

$$e(t_j^+) = f(t_j) - x_1(t_j),$$

Tj = tj+1 – tj = ψ [e(tj)], ti+1 = Ti + ti

3. Calculate the state variables by the relations (4)-(6).

4. At the same time, we calculate the number of closures l=k+1 of the pulse element that occurred in the interval (0,

θ], checking the condition ti ≤ θ

5. Starting from the (k+1) th step (t >θ, j = k+1, k+2,..., N), we define

$$e(t_j^+) = f(t_j) - x_1(t_j),$$

Tj = tj+1 – tj = ψ [e(tj)], Tj+1 = Tj + tj

6. If the conditions tj-k + θ < tj+1 ≤ tj-k + θ + τu,, are satisfied, then the state variables are calculated by the relations (7) – (9).

7. If the conditions tj-k + θ + τu < tj+1 ≤ tj-k+1 + θ, are satisfied, then the state variables are calculated by the relations (10) – (12).

Within the framework of the obtained results, multidimensional pulse-frequency systems with a delay can also be studied. Figure 6 shows a block diagram of an N-dimensional pulse-frequency system with delays in the controls.

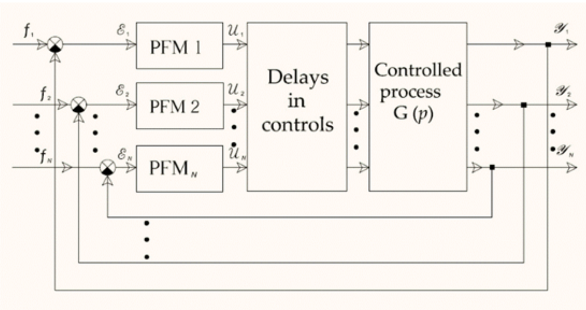

Figure 6. Multidimensional pulse-frequency system with control delays.

Autonomous pulse-frequency modulators implement the law of frequency modulation

Tjr = ψ r[erj(tjr)],

where tjr - the moment when the j-th pulse appears at the output of the r-th pulse-frequency modulator,

erj(tjr)- r - th channel error signal (r = 1, 2, ..., N).

Unsynchronized operation of autonomous modulators(in general), various delays in control actions cause significant difficulties in the study of the class of systems under consideration. Here, at first glance, there is an even more chaotic change of structural states than in a multidimensional pulse-width system with a delay. Nevertheless, by ordering the time sequences with which we are dealing in this case, it is possible to carry out a clear formalization of the description of the dynamics of the functioning of a multidimensional nonlinear discrete system with a constant delay.

Let the closing moments of the impulse elements tjr (r = 1, 2, ..., N) be determined at some j - th step. We obtain an unordered finite set of points:

T1 = (tj1, tj2, ..., tjN)

The set of time points that are delayed by the values θ rr, θ rk with respect to tjr, we denote:

$$T_2 = t_j^1 + \theta^{11}, t_j^1 + \theta^{21}, \dots, t_j^1 + \theta^{N1}, t_j^2 + \theta^{12}, t_j^2 + \theta^{22},$$

$$t_j^2 + \theta^{N2}, \dots, t_j^N + \theta^{1N}, t_j^N + \theta^{2N}, \dots, t_j^N + \theta^{NN})$$

We will set the set of control pulse termination moments at the outputs of the corresponding delay links:

$$T_3 = t_j^1 + \theta^{11} + \tau^1, t_j^1 + \theta^{21} + \tau^1, \dots, t_j^1 + \theta^{N1} + \tau^1, t_j^2 +$$

$$+\theta^{12} + \tau^2,$$

$$t_j^2 + \theta^{22} + \tau^2, \dots, t_j^2 + \theta^{N2} + \tau^2, \dots,$$

$$t_j^N + \theta^{1N} + \tau^N, t_j^N + \theta^{2N} + \tau^N, \dots, t_j^N + \theta^{NN} + \tau^N),$$

where θrr (r = 1, 2, ..., N) – delay in the r-th cross channel,

θrk (r = 1, 2, ..., N, k = 1, 2, ..., N, r≠k)) – the delay in the (rk) - th cross channel, τr - the duration of the frequency-modulated pulses urk.

Let's define the union of finite sets T1, T2, T3

$$T = T_1 \cup T_2 \cup T_3$$

Let's order all points of the set T using the numbering sequence 1,2, ..., n. On the time axis we get a finite set:

T*= (t1, t2 ,..., tn).

We will introduce the time interval into consideration t1 < t ≤ tn.

We will introduce the time interval into consideration t1 < t ≤ tn. Divide it into (n -1) intervals:

(t1, t2], (t2, t3], ..., (tn-1, tn].

Within each time interval (ti, ti+1], i= 1, 2, ..., n, a multidimensional pulse - frequency system with a delay is in the same structural state. Using a topological model, it is not difficult to determine the behavior of the system at all specified intervals, including moments: (t1, t2, ..., tn].

Performing the described procedure sequentially for each step of the calculations, (j = 0, 1, 2, ...,), it is possible to fully define the processes in a multidimensional pulse-frequency system with delays in the controls.

The dynamic graph model of a multidimensional multidimensional pulse-frequency system with delays in controls is constructed similarly to the graph model of a multidimensional pulse-width system with delays and is a combination of graph models of system elements:

$$G_t^a = G_t^f \cup G_t^{\varphi з} \cup G_t^z \cup G_t^{oy}.$$

Naturally, the calculation of such a structurally complex system as a multidimensional pulse-frequency system with a delay can only be performed on a computer.

The algorithm for calculating processes in such a system is proposed below.

Determination of state variables of a multidimensional system with pulse frequency modulation and control signal delay

of the system is constructed on the basis of combining graph models of its elements:

$$G_t^a = G_t^f \cup G_t^{\varphi з} \cup G_t^z \cup G_t^{oy}.$$

At each step of the calculations, j = 0, 1,..., m, in accordance with the modulation characteristics, we determine the closing moments of the pulse elements:

tj1,tj2,...,tjr,

We determine the moments of occurrence of control pulses at the outputs of the corresponding delay links:

$$t_j^1 + \theta^{11}, t_j^1 + \theta^{21}, \dots, t_j^1 + \theta^{N1},$$

$$t_j^2 + \theta^{12}, t_j^2 + \theta^{22}, \dots, t_j^2 + \theta^{N2},$$

... ...

$$t_j^N + \theta^{1N}, t_j^N + \theta^{2N}, \dots, t_j^N + \theta^{NN},$$

We determine the moments of the end of the delayed pulses

$$t_j^1 + \theta^{11} + \tau^1, t_j^1 + \theta^{21} + \tau^1, \dots, t_j^1 + \theta^{N1} + \tau^1,$$

$$t_j^2 + \theta^{12} + \tau^2, t_j^2 + \theta^{22} + \tau^2, \dots, t_j^2 + \theta^{N2} + \tau^2,$$

$$t_j^N + \theta^{1N} + \tau^N, t_j^N + \theta^{2N} + \tau^N, \dots, t_j^N + \theta^{NN} + \tau^N),$$

We arrange the obtained points on a single time scale and mark the interval δj on which they are located.

Using the graph, we determine the state variables, the output coordinates of the system for all time points marked on the interval δj.

7. Return to step 2.l of the algorithm.

Consider an example.

Let us calculate the transient process in a frequency-pulse system with an ideal frequency-pulse modulator (PFM) and with a control delay (Figure 7).

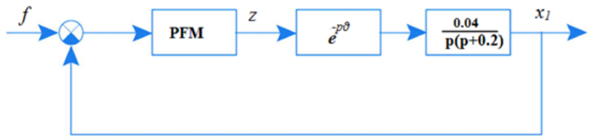

Figure 7. Structural diagram of a frequency-pulse system with an ideal frequency-pulse modulator (PFM) and with a control delay.

Initial data and system characteristics:

$$G(p) = \frac{0.04}{p(p+2)}; \; T_n = \frac{2.5}{0.1+|e_n|}; \; z_n = sign(e_n).$$

$$T_n = t_{n-1} - t_n; \quad \theta = 1 \text{ sek}$$

Solution:

$$e(t_0^+) = u(t_0) - x_1(t_0) = 1; z(t_0^+) = sign(e(t_0^+)) = 1;$$

$$T_0 = \frac{2.5}{0.1+|e(t_0^+)|} = 2.272;$$

$$t_1 = t_0 + T_0 = 2.272.$$

We build a graph of transition states of the system (Figure 8).

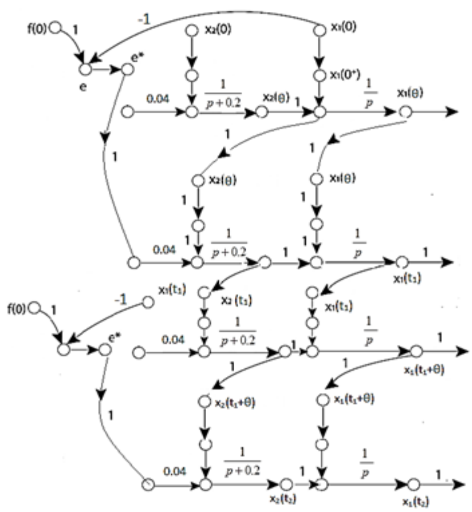

Figure 8. Dynamic graph model of one-dimensional nonlinear system.

with a pulse-frequency delay modulator.

Based on the graph model for the state variables of the system at the time $t = t_0 + \theta$, we write the following expressions:

$$x_2(\theta) = L^{-1}\left\{\frac{1}{p+0.2}\right\} x_2(t_0) = e^{-0.2\theta} x_2(t_0) = 0 \quad ;$$

$$x_1(\theta) = x_1(t_0) + L^{-1}\left\{\frac{1}{p(p+0.2)}\right\} x_2(t_0) =$$

$$= x_1(t_0) + 5(1 - e^{-0.2\theta}) x_2(t_0) = 0$$

$$x_2(t_1) = L^{-1}\left\{\frac{1}{p+0.2}\right\} [x_2(\theta) + 0.04 z(t_0^+)] =$$

$$= e^{-0.2(t_1-\theta)} [x_2(\theta) + 0.04 z(t_0^+)] = 0.031011;$$

$$x_1(t_1) = x_1(\theta) + L^{-1}\left\{\frac{1}{p(p+0.2)}\right\} [x_2(\theta) + 0.04 z(t_0^+)] =$$

$$= x_1(\theta) + 5(1 - e^{-0.2(t_1-\theta)}) [x_2(\theta) + 0.04 z(t_0^+)] = 0.044946$$

For system state variables at time $t = t_1 + \theta$ we write the following expressions:

$$x_2(t_1+\theta) = L^{-1}\left\{\frac{1}{p+0.2}\right\} [x_2(t_1) =$$

$$= e^{-0.2\theta} x_2(t_1) = 0.025389$$

$$x_1(t_1+\theta) = x_1(t_1) + L^{-1}\left\{\frac{1}{p(p+0.2)}\right\} x_2(t_1) =$$

$$= x_1(t_1) + 5(1 - e^{-0.2\theta}) x_2(t_1) = 0.073053$$

For system state variables at time $t = t_2$= 4,707 we write the following expressions:

$$x_2(t_2) = L^{-1}\left\{\frac{1}{p+0.2}\right\} [x_2(t_1+\theta) + 0.04 z(t_1^+)] =$$

$$= e^{-0.2(t_2-t_1-\theta)} [x_2(t_1+\theta) + 0.04 z(t_1^+)] = 0.049722$$

$$x_1(t_2) = x_1(t_1+\theta) + L^{-1}\left\{\frac{1}{p(p+0.2)}\right\} \times$$

$$\times \left[x_2(t_1+\theta) + 0.04 z(t_1^+)\right] =$$

$$= x_1(t_1+\theta) + 5(1 - e^{-0.2(t_2-t_1-\theta)}) \times$$

$$\times \left[x_2(t_1+\theta) + 0.04 z(t_1^+)\right] = 0.151389$$

For system state variables at time $t = t_2 + \theta$ we write the following expressions:

$$x_2(t_2+\theta) = L^{-1}\left\{\frac{1}{p+0.2}\right\} [x_2(t_2) = e^{-0.2\theta} x_2(t_2) = 0.040709$$

$$x_1(t_2+\theta) = x_1(t_2) + L^{-1}\left\{\frac{1}{p(p+0.2)}\right\} x_2(t_2) =$$

$$= x_1(t_1) + 5(1 - e^{-0.2\theta}) x_2(t_1) = 0.196455$$

In general, for time points $t_i + \theta, i = 0, 1, \dots, n$ expressions for system state variables will look like:

$$x_2(t_{i+1}) = L^{-1}\left\{\frac{1}{p+0.2}\right\} [x_2(t_i+\theta) + 0.04 z(t_i^+)] =$$

$$= e^{-0.2(t_{i+1}-t_i-\theta)} [x_2(t_i+\theta) + 0.04 z(t_i)];$$

$$x_1(t_{i+1}) = x_1(t_i+\theta) + L^{-1}\left\{\frac{1}{p(p+0.2)}\right\} \times$$

$$\times \left[x_2(t_i+\theta) + 0.04 z(t_i^+)\right] =$$

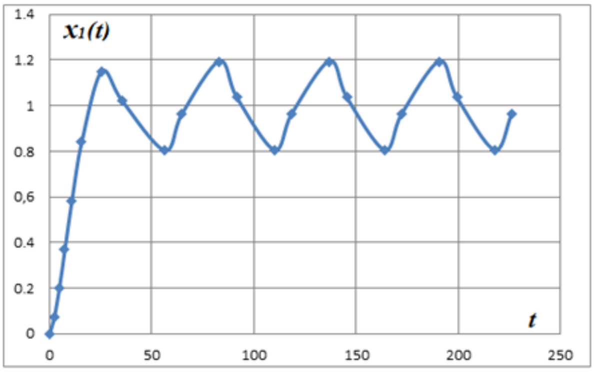

Figure 9. Graph of the transient process in the frequency-pulse system

system on the time interval t ∈ [0, 150] , obtained on the basis of the graph model.

Thus, we find successively the values of the system state variables x1(t), x2(t) at time t3, the error signal at the moment of the subsequent closing of the pulse element e(t3+), the output signal of the pulse-frequency modulator z(t3+), etc.

The calculation of the transient process in this system with PFM based on its graph model was performed in Microsoft Excel. Based on the results of the calculation, a graph of the output coordinate of the system was obtained - x1(t). On fig. 9 shows a graph of the transient process in the system on the time interval t ∈ [0, 150]

Conclusions. Dynamic graph models allow us to obtain recurrent relations for calculating processes in pulse-frequency systems with a delay. Based on the results obtained, algorithms for calculating processes in a one - dimensional and multidimensional pulse-frequency system with a control delay are obtained [20-22]. The use of transition state graphs makes it possible to perform a clear formalization and automation of computational procedures for calculating systems with non-program changes in the structure. The ordering of the time sequences that take place in this case allows us to perform a clear formalization of the dynamics of the structures of one-dimensional and multidimensional nonlinear systems with a delay.

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